

# A soft landing on probability spaces

MTH382 Probability Theory for Finance and Actuarial Science

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# What is inside ?

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- Around the idea of Probability theory.
- An informal definition of a probability space.
- Constructions of an event space.

# What is Probability theory ?

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**Probability theory** is a mathematical theory that helps dealing with uncertain nature of the world.

The uncertainty here may range from coin flipping, dice rolling, casino games, stock prices, accidents, weather conditions, human behaviors, and much more.

Being able to systematically and mathematically capture the randomness, probability theory is used in vast applications including finance, risk management, insurance, game theory, resource management, etc.

## An idea of a probability space

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To describe a *probability space*, we require three ingredients:

- a *sample space*  $\Omega$  containing all possible *outcomes*,
- a *event space*  $\mathcal{F} \subset 2^\Omega$  containing all observable *events*,
- a *probability measure*  $P : \mathcal{F} \rightarrow [0, 1]$ , assigning to each event a numerical value between 0 and 1 representing its probability.

Again, we insist that it might be *impossible* to assign a probability to every subset of  $\Omega$ . This behavior is typical in measure theory, but is also quintessential in probability theory.

*Example 1.*

Consider tossing two identical coins simultaneously and discuss that not every subset should be considered as an event.

*Example 2.*

From Example 4, what is the full event space ?

## Rolling dice (Odd – Even)

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*Example 3.*

Consider dice rolling and focus on the “Odd – Even” scenarios and write down its probability space.

## Rolling dice (Odd – Even – High – Low)

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### *Example 4.*

In this example, we consider the “Odd – Even – High – Low” cases for dice rolling.

Task. Write the probability space corresponding to the above scenarios.



## Don Juan getting married

### *Example 5.*

Don Juan has two girlfriends, Amy and Betty, and he is now considering marrying one of them. Being a womanizer, we could not decide who to marry so he plans to make a phone call to each of them. Here are the rules of this marriage:

- a. Don Juan makes the first call to one of the two girlfriends at random based on a coin toss.
- b. Each girlfriend does not know if she got the first or second call.
- c. Each girlfriend either accept or reject the proposal at random based on a coin toss.
- d. If the girlfriend of the first call accepted the proposal, they get married and live happily everafter.
- e. If the girlfriend of the first call rejected the proposal, then Don Juan makes the second call to the other girlfriend.

Task. Write the probability space in views of Don Juan, Amy, and Betty.

## Takeaways

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- Events are the subsets of  $\Omega$  where the probabilities could be assigned.
- Not all subsets of  $\Omega$  is an event. Hence not all subsets is assigned with a probability.
- We start the construction of an event space by listing base events, then forming the secondary events by (countable) complements, unions, and intersections.
- Different people perceives different event spaces, and hence probability spaces.

