Optimization Modeling — Part 2/4

Deterministic models

Parin Chaipunya
KMUTT
└ Mathematics @ Faculty of Science
└ The Joint Graduate School of Energy and Environments

Areas of research:

- Multi-agent optimization: Bilevel programs, Game theory
- Optimization modeling: mainly focused on energy and environmental applications

A coordinated course with Metropolitan Electricity Authority (MEA) on STOCHASTIC OPTIMIZATION

by Parin CHAIPUNYA (KMUTT, Thailand) and Michel DE LARA (ENPC, France)

17 Nov – 18 Dec 2025

การไฟฟ้านครหลวง
Metropolitan Electricity Authority

KMUTT

CERMICS
École nationale des ponts et chaussées

parin.cha@kmutt.ac.th
parinchaipunya.com
github.com/parinchaipunya

Overview

The main objective of this lecture is to get you to know the **framework and workflow of optimization and modeling**.

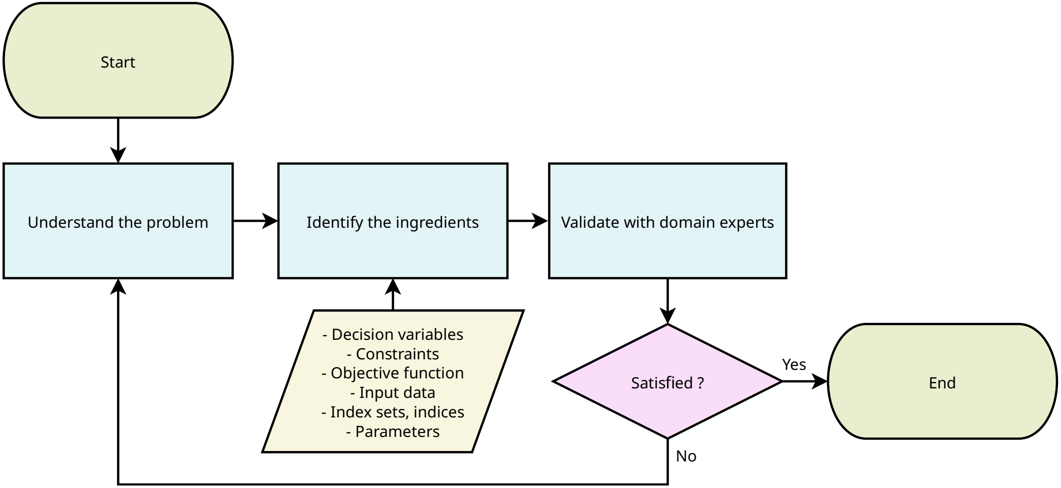
Table of contents

What is optimization modeling ?

"Modeling is communication."

Optimization modeling is the process of turning a real-life decision problem into a solvable mathematical program.

Modeling workflow



Common modeling mistakes

- Using or putting together wrong units (kW vs MW vs MWh).
- Missing some constraints (e.g., ramping, buffer time)
- Choosing objective that does not reflect the true goal.
- Forgetting integer decisions.
- Forgetting domain bounds.
- Over-simplifying.
- Writing nonlinear constraints when a linear equivalent exists.

Modeling checklist

- List all ingredients
- Ensure variable units are consistent
- Write a tiny numerical example (mini-case)
- Check the solution for integrity
- Make a table/visualization summarizing everything

Section 2

Modeling examples/exercises

Subsection 1

Single time-step economic dispatch

Example 1 — Single time-step economic dispatch

Story
A system operator must supply a known demand at a single moment (*e.g.*, one hour). There are several generation technologies available:

- gas
- coal
- peaker

Each technology has:

- a cost per MWh,
- a maximum production capacity,

We want to meet the demand at minimum cost.

Example 1 — Single time-step economic dispatch

To do the modeling, first we fill up the following table.

Notation	Quantifier	Description
Sets		
I		The set of technologies, indexed with $i \in I$.
Parameters (Data)		
D		The known demand (MW).
C_i	$\forall i \in I$	Cost per MWh of technology i.
$P_i^{\max}$	$\forall i \in I$	Maximum capacity (MW).
Decision variables		
$u_i \geq 0$	$\forall i \in I$	How much the technology i outputs (MW).
Constraints		
$\sum_{i \in I} u_i = D$		The production meets exactly the demand.
$0 \leq u_i \leq P_i^{\max}$	$\forall i \in I$	Capacity limits of each technology.
Objective function		
$\sum_{i \in I} C_i u_i \leftarrow$ Minimize		The total generation cost summing up $C_i u_i = \underbrace{C_i}_{\text{cost per MWh}} \times \underbrace{u_i}_{\text{power output}} \times \underbrace{1}_{\text{one hour}}$

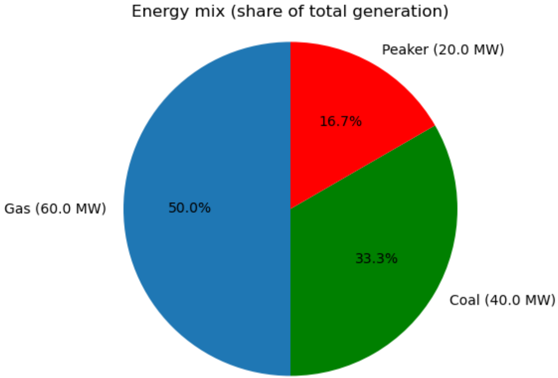
Example 1 — Single time-step economic dispatch

Once we finished, the parameters section tells us the data we need for the model. In this case, the data is provided in the following table.

Technology	**Cost (€/MWh)**	**Maximum capacity (MW)**	**Demand (MW)**
Gas	20	60	
Coal	40	40	120
Peaker	120	100	

Example 1 — Single time-step economic dispatch

Once the model is solved, we get this optimal energy mix.



Subsection 2

Multiple time-step economic dispatch (independent case)

Example 2 — Multiple time-step economic dispatch (independent case)

In this section, we make the case of Example 1 more complicated.

Story
A system operator must supply a known demand during each 60 minutes window for a day.
There are several generation technologies available:

- gas
- coal
- peaker
- solar pv (supposed that the solar availability is known)

Each technology has:

- a cost per MWh,
- a maximum production capacity,

We want to meet the demand at minimum cost.

Example 2 — Multiple time-step economic dispatch (independent case)

Notation	Quantifier	Description
Sets		
I		The set of technologies excluding solar pv, indexed with $i \in I$.
T		The set of time-steps, indexed with $t \in T$.
Parameters (Data)		
Δt		The duration of between time-steps.
D_t	$\forall t \in T$	The known demand at time t (MW).
C_i	$\forall i \in I$	Cost per MWh of technology i.
$P_i^{\max}$	$\forall i \in I$	Maximum capacity (MW).
S_t	$\forall t \in T$	Available solar pv power (MW) at the time-step t (probably from the forecase).

Example 2 — Multiple time-step economic dispatch (independent case)

Notation	Quantifier	Description
Decision variables		
$u_{i,t} \geq 0$	$\forall i \in I,\ \forall t \in T$	How much the technology i outputs during the time-step t (MW).
$u_t^{\text{pv}} \geq 0$	$\forall t \in T$	Solar pv power injected durint the time-step t (MW).
Constraints		
$\sum_{i \in I} u_{i,t} + u_t^{\text{pv}} = D_t$	$\forall t \in T$	The production meets exactly the demand at each time-step t.
$0 \leq u_{i,t} \leq P_i^{\max}$	$\forall i \in I,\ \forall t \in T$	Capacity limits of each technology.
$0 \leq u_t^{\text{pv}} \leq S_t$	$\forall t \in T$	Capacity limit of solar pv.
Objective function		
$\sum_{i \in I} C_i u_i \Delta t \leftarrow$ Minimize		The total production cost.

Example 2 — Multiple time-step economic dispatch (independent case)

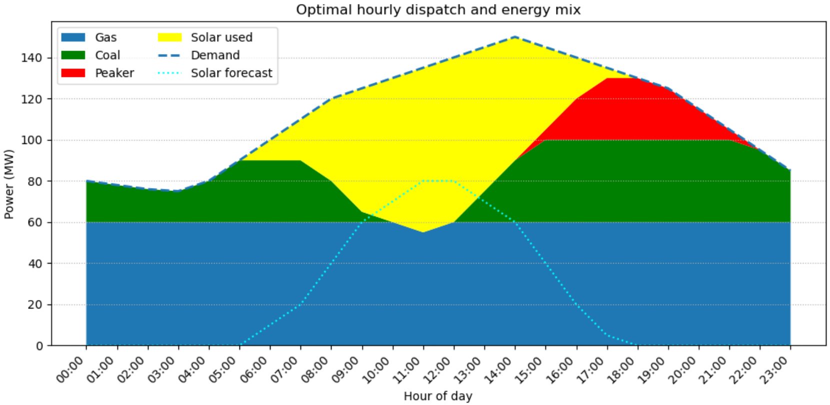
We now need some data, the table earlier and also the hourly demand and solar pv forecast.

Technology	Cost (€/MWh)	Maximum capacity (MW)
Gas	20	60
Coal	40	40
Peaker	120	100

Hour	Demand (MW)	Solar forecast (MW)
00:00	80	0
01:00	78	0
02:00	76	0
03:00	75	0
04:00	80	0
05:00	90	0
06:00	100	10
07:00	110	20
08:00	120	40
09:00	125	60
10:00	130	70
11:00	135	80
12:00	140	80
13:00	145	70
14:00	150	60
15:00	145	40
16:00	140	20
17:00	135	5
18:00	130	0
19:00	125	0
20:00	115	0
21:00	105	0
22:00	95	0
23:00	85	0

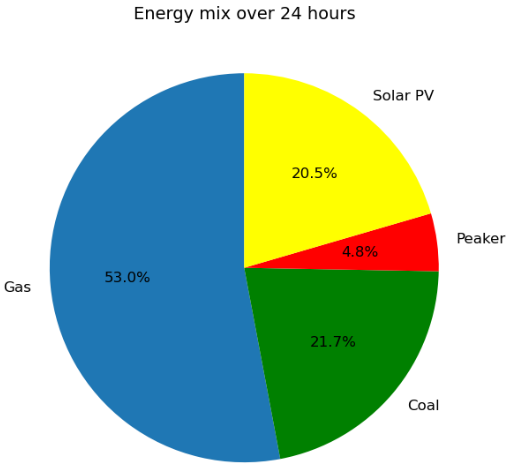
Example 2 — Multiple time-step economic dispatch (independent case)

Once the model is solved, we get this optimal energy dispatch.



Example 2 — Multiple time-step economic dispatch (independent case)

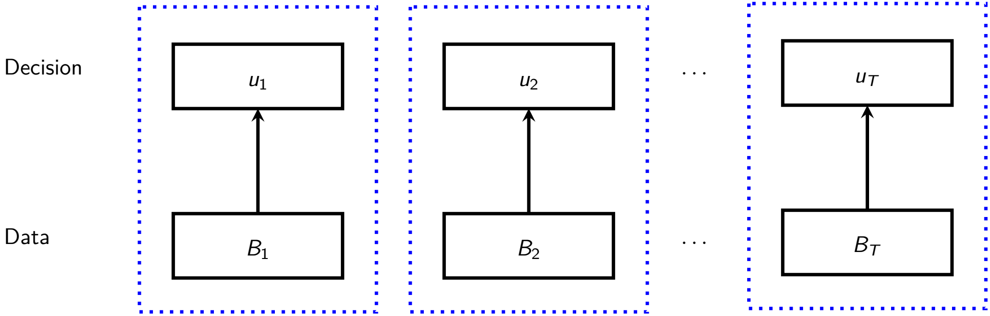
Once the model is solved, we get this optimal energy mix.



Example 2 — Multiple time-step economic dispatch (independent case)

The characteristic of this decision model is that the decisions are time-independent, *i.e.*, decision at a time-step t has no influence on the future time-step $t+1$.

This behavior is illustrated in the following diagram, where each blue dotted box is actually an independent problem.



Subsection 3

Multiple time-step economic dispatch (dynamic case)

Example 3 — Multiple time-step economic dispatch (dynamic case)

In this example, we need to introduce an additional ingredient — the **states**.

We write generically a **state space** $\mathcal{X}$, with the state element $x \in \mathcal{X}$.

Typically, a **current state** $x \in \mathcal{X}$ and a **decision** $u \in \mathcal{U}$ together bring us into a new state through a **dynamical equation**

$$x^+ = f(x, u),$$

where $f : \mathcal{X} \times \mathcal{U} \to \mathcal{X}$ is the operator that describes this dynamics.

We may take the battery level as an example. In this case, the state $x \in \mathcal{X} = \mathbb{R}$ is the battery level, $(u_{\text{charge}}, u_{\text{discharge}}) \in \mathcal{U} = \mathbb{R}^2$ is the decided amounts of charge and discharge and

$$x^+ = f(x, u) = x + u_{\text{charge}} - u_{\text{discharge}}$$

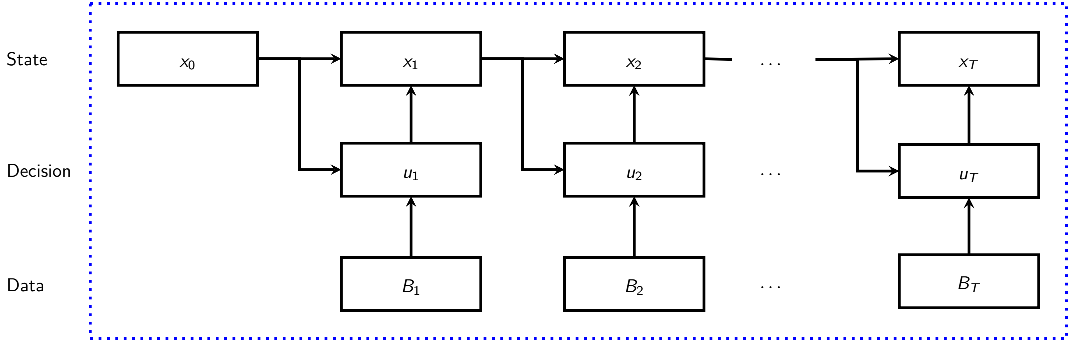
describes the dynamics.

Example 3 — Multiple time-step economic dispatch (dynamic case)

In this example, we shall explore the case where a decision at time-step t alters some states that affects a decision at $t+1$.

Here the decision u_{t+1} is made with the knowledge of the state x_t, and the state x_{t+1} is updated with a dynamical equation $x_{t+1} = f(x_t, u_t)$.

This is best illustrated by the following diagram.



Example 3 — Multiple time-step economic dispatch (dynamic case)

Story

A system operator must supply a known demand during each 60 minutes window for a day.
There are several technologies available:

- gas
- coal
- peaker
- solar pv (supposed that the solar availability is known)
- a battery unit.

Each technology has:

- a cost per MWh,
- a maximum capacity,
- a charge/discharge capacity and efficiency for the battery.

We want to meet the demand at minimum cost.

Example 3 — Multiple time-step economic dispatch (dynamic case)

Notation	Quantifier	Description
Sets		
I		The set of technologies excluding solar pv, indexed with $i \in I$.
T		The set of time-steps, indexed with $t \in T$.
Parameters (Data)		
Δt		The duration of between time-steps.
D_t	$\forall t \in T$	The known demand at time t (MW).
C_i	$\forall i \in I$	Cost per MWh of technology i.
$P_i^{\max}$	$\forall i \in I$	Maximum capacity (MW).
S_t	$\forall t \in T$	Available solar pv power (MW) at the time-step t.
x_0		The initial energy state of the battery (MWh).
$x^{\max}$		The maximum energy capacity of the battery (MWh).
$P_+^{\max}$		The maximum charging power of the battery (MW).
$P_-^{\max}$		The maximum discharging power of the battery (MW).
$\eta_+ \in [0,1]$		The charging efficiency of the battery.
$\eta_- \in [0,1]$		The discharging efficiency of the battery.

Example 3 — Multiple time-step economic dispatch (dynamic case)

Notation	Quantifier	Description
Decision and state variables		
$u_{i,t} \geq 0$	$\forall i \in I,\ \forall t \in T$	How much the technology i outputs during the time-step t (MW).
$u_t^{\text{pv}} \geq 0$	$\forall t \in T$	Solar pv power injected durint the time-step t (MW).
$u_t^{+} \geq 0$	$\forall t \in T$	Charging power of the battery during the time-step t (MW).
$u_t^{-} \geq 0$	$\forall t \in T$	Discharging power of the battery during the time-step t (MW).
$x_t \geq 0$	$\forall t \in T$	The state of charge of the battery at the time-step t (MWh).

Example 3 — Multiple time-step economic dispatch (dynamic case)

Notation	Quantifier	Description
Constraints		
$\sum_{i\in I} u_{i,t} + u_t^{\text{pv}} + u_t^- = D_t + u_t^+$	$\forall t \in T$	Power balance at each time-step t.
$0 \leq u_{i,t} \leq P_i^{\max}$	$\forall i \in I,\ \forall t \in T$	Capacity limits of each technology.
$0 \leq u_t^{\text{pv}} \leq S_t$	$\forall t \in T$	Capacity limit of solar pv.
$0 \leq u_t^+ \leq P_+^{\max}$	$\forall t \in T$	The charging capacity at each time-step t.
$0 \leq u_t^- \leq P_-^{\max}$	$\forall t \in T$	The discharging capacity at each time-step t.
$0 \leq x_t \leq x^{\max}$	$\forall t \in T$	Energy capacity limit at each time-step t.
$x_t = x_{t-1} + \eta_+ u_t^+ \Delta t - \frac{1}{\eta_-} u_t^- \Delta t$	$\forall t \in T$	The battery's state-of-charge dynamical equation.
Objective function		
$\sum_{i\in I} C_i u_i \Delta t \leftarrow$ Minimize		The total production cost.

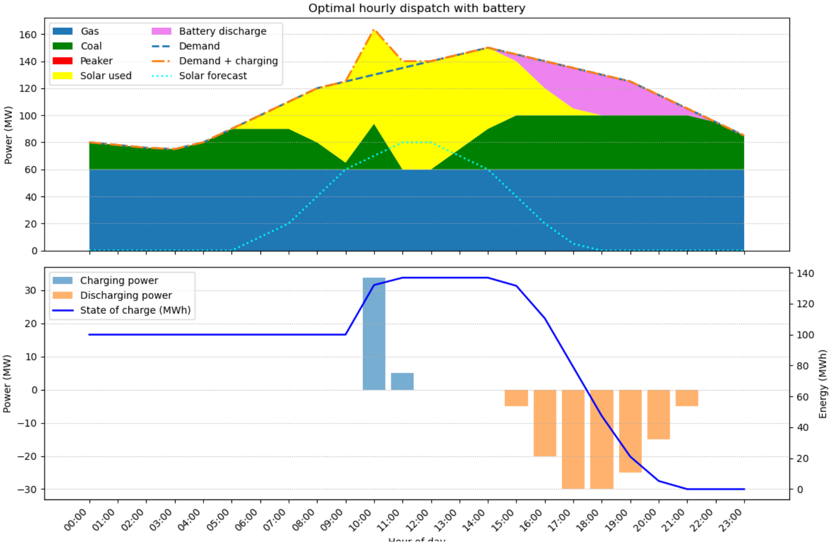
Example 3 — Multiple time-step economic dispatch (dynamic case)

In addition to the data required in Example 2, we need the following battery data.

Parameter	**Value**
x^{max}	200 MWh
P_{+}^{max}	50 MW
P_{-}^{max}	50 MW
η_{+}	0.95
η_{-}	0.95
x_0	100 MWh

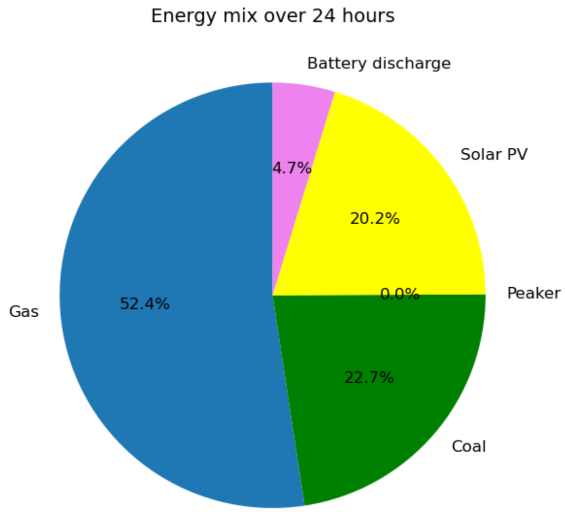
Example 3 — Multiple time-step economic dispatch (dynamic case)

The following is the visualization of the optimal economic dispatch.



Example 3 — Multiple time-step economic dispatch (dynamic case)

The following is the visualization of the optimal economic dispatch.



Subsection 4

Transmission line expansion

Example 4 — Transmission line expansion (DC power flow)

Story

- Some lines are now frequent bottlenecks, forcing power to take longer or less efficient paths and, in extreme cases, causing unavoidable load shedding when flows cannot reach certain regions.
- Building new lines can relieve these bottlenecks, but each project is costly and must be justified by tangible operational improvements.
- The operator needs a systematic way to decide which candidate lines to build, balancing investment costs against gains in reliability, reduced congestion, and avoided load shedding.

Example 4 — Transmission line expansion (DC power flow)

Notation	Quantifier	Description
Sets		
N		Set of buses, indexed by i and j.
L^{ex}		Set of existing transmission lines, indexed by ℓ
L^{cand}		Set of candidate transmission lines, indexed by k
Parameters		
d_i	$\forall i \in N$	Demand at bus i (MW).
g_i	$\forall i \in N$	Net generation at bus i (MW).
c_k^{inv}	$\forall k \in L^{\text{cand}}$	Investment cost of line k.
c_i^{shed}	$\forall i \in N$	Penalty cost of load shedding.
$\overline{F}_\ell$	$\forall \ell \in L^{\text{ex}}$	Capacity of existing line ℓ (MW).
$\overline{F}_k^{\text{new}}$	$\forall k \in L^{\text{cand}}$	Capacity of candidate line k (MW).
B_ℓ	$\forall \ell \in L^{\text{ex}}$	Susceptance of existing line ℓ.
B_k^{new}	$\forall k \in L^{\text{cand}}$	Susceptance of candidate line k.
$r = 1$		Reference bus for angle fixation.

Example 4 — Transmission line expansion (DC power flow)

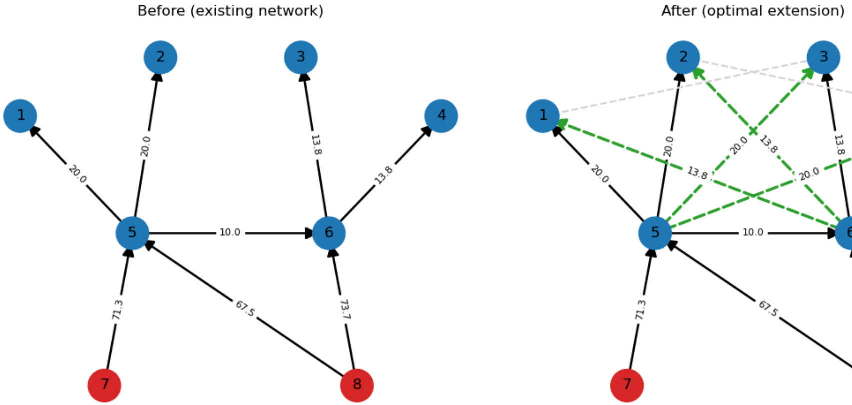
Notation	Quantifier	Description
Decision variables		
$u_i^{\theta} \in \mathbb{R}$	$\forall i \in N$	Voltage angle at bus i (rad).
$u_\ell^{f} \in \mathbb{R}$	$\forall \ell \in L^{\text{ex}}$	Power flow on existing line ℓ (MW).
$u_k^{f,\text{new}} \in \mathbb{R}$	$\forall k \in L^{\text{cand}}$	Power flow on candidate line k (MW).
$u_i^{\text{shed}} \in \mathbb{R}$	$\forall i \in N$	Load shedding (> 0) /spilling (< 0) at bus i (MW).
$u_k^{\text{inv}} \in \{0, 1\}$	$\forall k \in L^{\text{cand}}$	1 if line k is built.

Example 4 — Transmission line expansion (DC power flow)

Notation	Quantifier	Description
Constraints		
$g_i - d_i + u_i^{\text{shed}} = \sum_{\ell=(i,j)} u_\ell^f - \sum_{\ell=(j,i)} u_\ell^f + \sum_{k=(i,j)} u_k^{f,\text{new}} - \sum_{k=(j,i)} u_k^{f,\text{new}}$	$\forall i \in N$	Power balance.
$u_\ell^f = B_\ell \left(u_i^\theta - u_j^\theta\right)$	$\forall \ell = (i,j)$	DC flow on existing lines.
$u_k^{f,\text{new}} = B_k^{\text{new}} \left(u_i^\theta - u_j^\theta\right)$	$\forall k = (i,j)$	DC flow on candidate lines.
$0 \leq u_\ell^f \leq \overline{F}_\ell$	$\forall \ell \in L^{\text{ex}}$	Capacity of existing lines.
$0 \leq u_k^{f,\text{new}} \leq \overline{F}_k^{\text{new}} u_k^{\text{inv}}$	$\forall k \in L^{\text{cand}}$	Capacity of candidate lines.
$u_r^\theta = 0$		Reference bus angle.
Objective function		
$\sum_{k \in L^{\text{cand}}} c_k^{\text{inv}} u_k^{\text{inv}} + \sum_{i \in N} c_i^{\text{shed}} (u_i^{\text{shed}})^2 \leftarrow$ Minimize		Minimize investment + quadratic penalty.

Example 4 — Transmission line expansion (DC power flow)

The optimal expansion can be illustrated as follows.



Section 3

Conclusion

Key takeaways

- Optimization is a **framework** for turning decision problems into mathematics.
- Modeling is a **workflow**, and it usually requires iterating many times through the processes.
- Start simple and clean, then add complexity.
- A mini-case is usually useful especially in a large-scape problem.
- Always check the results and verify if it is physically realistic.

What's more ?

- Nonlinear and nonconvex programs (reformulate, simplify, or face it)
- Robustness (sensitivity analysis)
- Uncertainty (stochastic modeling)

-» Continue to **Part 3**.